Shadow Pricing Ecosystem Services in Boreal Forests

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ABSTRACT

Boreal forests jointly produce marketed goods and non-market ecosystem services, generating unavoidable trade-offs. We estimate shadow prices with the directional distance function (R. G. Chambers et al. 1996), endogenizing the directional vector and correcting for sampling variation via bootstrap methods. Using a county-by-year panel of Swedish forests, we find that shadow prices vary spatially and temporally yet remain generally low, implying modest opportunity costs for improving biodiversity and carbon sequestration relative to timber. Morishima elasticities further characterize substitution possibilities and least-cost paths to better environmental outcomes. The results indicate unexploited scope for conservation policy in boreal systems.

Keywords: Forest Ecosystem Services; Shadow Pricing; Directional Distance Function; Morishima Elasticity

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1. Introduction

Boreal forests, located in the Earth's northern latitudes, comprise one-third of global forest cover and constitute the largest terrestrial biome (UNEP et al. 2009). Boreal forests hold immense potential for marketed wood production and play an important role in regional economies. In Sweden, the forest sector contributes approximately SEK 145–185 billion annually in export revenues (around 10% of total goods exports and 2.5% of GDP) and directly employs over 100,000 people, particularly in rural and northern regions where alternative economic activities are limited.[i] They also provide critical ecosystem services, including biodiversity and carbon sequestration, which support human well-being more broadly.

The European Union framework, *The New EU Forest Strategy for 2030*[ii], highlights the importance of balancing environmental, economic, and social functions of forests, including their

role in mitigating and adapting to climate change. The framework emphasizes ecosystem services associated with forests, such as the provisioning of wood, non-wood products and food, bioenergy, carbon sequestration, biodiversity, clean air, and water. In addition, wood-based products contribute to climate change mitigation by sequestering carbon and serving as alternatives to commodities with more substantial carbon footprints, including building and packaging materials, textiles, chemicals, and fuels. Wood-based products are largely renewable and recyclable, two key aspects of circular bio-economies. This underscores the forestry sector and forest-based industries as essential stakeholders in advancing sustainable economic growth.

In practice, sustainable forest management can entail both trade-offs and synergies between these various ecosystem services and commercial forest production. We examine these for the case of Sweden, which is roughly 70% forested, with 80% of forest land in active use[1]. Swedish Forest management decisions inherently involve land-use trade-offs among competing objectives such as timber production, bioenergy supply, carbon storage and biodiversity conservation. Swedish forest management policy also seeks to achieve national and EU sustainability goals, while incorporating local stakeholders who are often more directly affected by forest management decisions.

We adopt a production theoretical framework to model the joint production of commercial forest products and non-marketed forest ecosystem services. We use the directional distance function (DDF) model (R. G. Chambers et al. 1996; 1998) to represent the forest production technology, and then exploit duality theory to recover the corresponding trade-offs between marketed and non-marketed goods and services in terms of their respective shadow prices. These shadow prices measure production-side opportunity costs, namely, the foregone marketed revenue associated with marginal changes in ecosystem service provision, rather than social willingness-

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to-pay or welfare-based valuations. We also compute Morishima Elasticities (Morishima 1967) to examine the sensitivity of substitution and complementarity between the various forest goods and services. This better takes into account the ecological complexity of forest production systems. Framed in this way, the shadow prices inform the least-cost design of production and conservation incentives within the existing technological frontier.

Our work makes several contributions to the forest ecosystem services literature, as well as the production-based valuation literature more broadly. First, we consider multiple goods and services simultaneously, as opposed to a single marketed good and a single non-marketed good, as in the majority of previous shadow price studies. These goods and services include marketed timber and biofuel production, as well as non-marketed carbon sequestration and biodiversity. Second, we model these trade-offs over many years, in a panel production model, rather than for a single point in time. Our panel model also allows for spatial heterogeneity in forest production systems, across the 21 counties of Sweden. Third, we combine advanced procedures for both optimization and statistical inference in our empirical estimation of the production technology (Färe et al. 2017; Leopold Simar and Wilson 1998; 2000). Finally, our application is novel. To the best of our knowledge, we are the first to value trade-offs between ecosystem services and commercial forest products in the context of boreal forests.

We find average county-level shadow prices for biodiversity and carbon sequestration of SEK 3 per m^3 of deadwood and SEK 50 per ton of carbon sequestered, respectively, which are notably lower than those reported in previous studies. In contrast, the average shadow price of carbon emissions is approximately SEK 2000 per ton, exceeding the current EU ETS price. Our results also vary by region, suggesting the need for more targeted incentives for forest owners to prioritize ES provision and carbon sequestration over shorter-term economic gains.

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The remainder of this paper is organized as follows: Section 2 reviews non-marketed valuation in the forestry sector and the application of distance function in determining shadow prices. Section 3 specifies the technology and methodology. Section 4 describes data and exhibits bias-corrected statistics for the estimates, and we present Morishima elasticities among the outputs. In section 5, we discuss the complexity of forest production and policy implications to finalize the paper.

2. Literature Review

Valuing Forest Ecosystem Services

The term ecosystem services (ES) broadly refers to the benefits to people generated by ecosystems, and is conventionally classified into four categories: (1) production and provisioning services, which primarily include marketed goods, such as timber, bio-based pharmaceuticals, foods and energy resources; (2) Regulating services, which include carbon sequestration and flood control; (3) Habitat and supporting services, which contribute to biodiversity; (4) Recreational and cultural services, which also include aesthetics (Millennium ecosystem assessment 2005; De Groot et al. 2012; Chen et al. 2016).

The majority of forest ES, beyond production and and provisioning services, lack market prices and must instead rely on non-market valuation methods for any associated welfare considerations. The forest management literature on valuing ES is extensive and can broadly be categorized into two main streams. The first stream values ES through their direct or potential market benefits and costs, such as expenditure-related approaches (A. Freeman III 1979; Sorg et al. 1984), damage function approaches (Krieger 2001; A. M. Freeman III et al. 2014) and contingent valuation (Mitchell et al. 1989; Beinat and Nijkamp 1998; Linkov and Moberg 2011). The second stream values ES in terms of the physical trade-offs between marketed timber

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production and related ES outcomes, mainly employing biophysical modeling (Hartman 1976; Amacher et al. 2009). See Nalle et al. (2004), Steffen et al. (2015) and Duncker et al. (2012) for trade-offs concerning biodiversity preservation, and Pussinen et al. (2002), Garcia-Gonzalo et al. (2007), Duncker et al. (2012), Pyörälä et al. (2014) for carbon sequestration, as well as Duncker et al. (2012) for water quality regulation.

Estimated forest ES values range widely in the literature, due primarily to the choice of valuation method, the specific ecosystem service being valued, and the spatial and temporal context of the study. We summarize this range for our two key ESs of interest, biodiversity and carbon sequestration, in Appendix Table A1. For example, a contingent valuation study in the UK estimated willingness to pay for biodiversity protection at € 0.45–3.89 per year (Garrod and Willis 1997). This highlights the relatively modest monetary values that households attach to broad biodiversity conservation. In contrast, global estimates of the social cost of carbon suggest much higher values, with Thorsen et al. (2014) reporting a minimum of € 61.25 per ton. This underscores the significant economic weight placed on carbon abatement compared to other forest ecosystem services.

Given the broad and context-specific nature of the term biodiversity, particularly in the context of forests, the definition of biodiversity can also vary significantly across studies. For instance, Nijkamp et al. (2008) review the different forms of forest biodiversity that have been studied in European countries, including both direct and indirect use values. They show that the estimated value of forest landscapes in Sweden is € 14.1 per person per year using the CV method, and Norwegians have expressed a willingness to pay up to € 120.9 per person per year for the conservation of endangered species associated with forest ecosystems. We note that while the biodiversity values in Appendix Table A1 cannot be directly compared, due to differences in

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measurement and definition, they do provide a more general range of biodiversity values for reference.

Carbon sequestration, on the other hand, has a more precise definition and standard unit of measure, as well as a more direct connection to market prices. Carbon markets, such as the European Union Emissions Trading System (EU ETS), provide direct market-based estimates of the value of carbon sequestration. Additionally, mechanisms like the ETS Clean Development Mechanism (CDM) and the Biocarbon Fund enable developing countries to generate revenue from carbon credits, further highlighting the economic value of carbon sinks (Cavatassi 2005). That said, forest carbon sequestration value estimates also range widely in the literature, as presented in Appendix Table A1, panel (b).

Productivity-Based Approaches to Valuing ES

Stated-preference methods recover welfare-based measures grounded in individuals' willingness to pay, they do not explicitly account for the technological trade-offs embedded in forest production systems. In contrast, production-oriented shadow pricing derives marginal rates of transformation within the production frontier and thus measures opportunity costs from the supply side. These approaches address distinct but complementary dimensions of ecosystem service valuation.

Carbon markets provide an explicit market-based price for carbon emissions, but forest-related emissions are typically excluded because bioenergy emissions are treated as carbon-neutral and the forestry sector lies outside the scope of the EU ETS. Hence, our interest in shadow prices, which provide an implicit price for carbon emissions, in terms of the value of their associated trade-offs in the production process. Most of the production-oriented shadow price literature on

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carbon emissions, and pollution more generally, focuses on high-pollution industries, such as the manufacturing and energy sectors (Coggins and Swinton 1996; Lee et al. 2002; Bonilla et al. 2018; Badau and Rada 2022). P. Zhou et al. (2014) provide an early review of this literature.

While the majority of applications focus on undesirable outputs, such as pollution, the production-oriented shadow price framework can also be applied to desirable non-marketed environmental characteristics. To our knowledge, the first to do so is Färe et al. (2001), who derive shadow prices for public land conservation. In a later study, Bostian and Herlihy (2014) derive shadow prices for various ecological indicators of wetland condition, also bridging environmental science. Building on their work, numerous studies (Khataza et al. 2017; Arjan Ruijs et al. 2017; Susaeta et al. 2019) use ecological indicators to quantify trade-offs between environmental characteristics, as well as the potential for substitution between both environmental and marketed outputs (Li et al. 2020). To our knowledge, we are the first to apply this framework to value tradeoffs between desirable ecosystem services and marketed output in the boreal forest context.

By reviewing previous studies, we find that the concept of ecosystem services in boreal forests has been treated in a rather broad manner, with substantial variation in both measurements and valuation approaches. In particular, the specific services of boreal forest biodiversity and carbon sequestration have received limited explicit attention in the existing literature. The vast majority of studies have treated ecosystem services as non-marketed outputs and evaluated them through contingent valuation methods and choice modeling. These methods lack a unified framework, relative to shadow pricing, for measuring trade-offs across heterogeneous ecosystem services, particularly when valuations must be anchored to a marketed good.

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3. Methodology

Theoretical Model

We define the boreal forest production technology that includes both desirable and undesirable outputs, $y = (y_1, y_2, \ldots, y_M) \in \mathbb{R}_+^M$ and $b = (b_1, b_2, \ldots, b_J) \in \mathbb{R}_+^J$, as well as inputs, $x = (x_1, x_2, \ldots, x_N) \in \mathbb{R}_+^N$. Taking an output orientation in order to model tradeoffs between y and b, we define the feasible output set, $P(x)$, as

$$P(x) = \{(y, b): x \text{ can produce } (y, b)\} \quad [1]$$

Our novel forest technology should satisfy the following properties in accordance to Färe et al. (2005, 2006):

- $P(\mathbf{x})$ is a compact set with $P(0) = \{0,0\}$. This means that the set is closed and bounded: $P(\mathbf{x})$ should be by nature a closed set and has a boundary, which indicates that, given a fixed level of inputs, it is not possible to produce unlimited outputs.
- Input is being freely (strongly) disposable. This assumes that producers have flexibility in utilizing inputs without output consequences. Mathematically, $x' \geq x$, implies $P(x') \supseteq P(x)$.
- Desirable output y and undesirable output b are weakly disposable such that proportional reductions of the outputs are feasible. Put this formally, $(y, b) \in P(\mathbf{x})$ and $0 \leq \theta \leq 1$ implies $(\theta y, \theta b) \in P(\mathbf{x})$. It ensures that it is costly to reduce the undesirables in the sense that it is inevitable to reduce the desirables at the margin such that (y, b) is feasible. It is formally implemented by the assumption that desirable outputs are by themselves freely disposable, such that if $(y, b) \in P(\mathbf{x})$, then for $y' \leq y$, $(y', b) \in P(\mathbf{x})$.

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We introduce the directional distance function (DDF) (R. G. Chambers 1998) as a complete representation of the technology (Fare et al. 2019). Our forest production process can be represented by an output-oriented DDF:

$$\vec{D}_o(x, y, b; \mathbf{g}) = \max\{\beta \geq 0: (y + \beta g_y, b - \beta g_b) \in P(\mathbf{x})\}, \quad [2]$$

where $\mathbf{g} = (g_y, g_b)$ is a directional vector that is informative on the path that maximizes the desirable output expansion and undesirable output contraction. $\vec{D}_o$ intuitively measures the distance between each observation to the production frontier, thus it is an efficiency measure. $\vec{D}_o$ equals 0 indicates that the production unit is on the frontier and is fully efficient, and the corresponding efficiency score is 1. A larger distance estimate indicates a higher inefficiency. Following Färe et al. (2005), the DDF inherits the properties of the production frontier and is subject to the axiomatic properties as we show in Appendix Table A2.

To value the tradeoffs between outputs, we exploit the duality relationship between the directional output distance function and maximal revenue (R. G. Chambers et al. 1998). For desirable output prices, $p = (p_1, p_2, \ldots, p_M)$ and undesirable output prices, $q = (q_1, q_2, \ldots, q_J)$, we define the maximal revenue function as,

$$R(x, p, q) = \max_{y,b}\{py - qb: \vec{D}_o(x, y, b; g) \geq 0\}. \quad [3]$$

By definition, $R(x, p, q) \geq (py - qb)$ for all feasible $(y, b) \in P(x)$, including $(y + \vec{D}_o(x, y, b; g)g_y,\ b - \vec{D}_o(x, y, b; g)g_b)$. Thus,

$$\vec{D}_o(x, y, b; \mathbf{g}) \leq \frac{R(x,p,q) - (py - qb)}{pg_y + qg_b}, \quad [4]$$

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which allows us to recover the directional output distance function, measured in primal quantity space, from the revenue function, measured in dual price space, as

$$\vec{D}_o(x, y, b; \mathbf{g}) = \inf_{p,q} \frac{R(x,p,q) - (py - qb)}{pg_y + qg_b}. \quad [5]$$

Applying the envelope theorem to [5] yields the output shadow prices,

$$\nabla_y \vec{D}_o(x, y, b; \mathbf{g}) = \frac{p}{pg_y + qg_b}, \quad [6a]$$

$$\nabla_b \vec{D}_o(x, y, b; \mathbf{g}) = -\frac{q}{pg_y + qg_b}. \quad [6b]$$

As long as at least one of the desirable outputs, $y_{m\prime}$ is also marketed, with market price, $p_{m\prime}$, we can recover the shadow prices of the other outputs, both desirable and undesirable, in monetary value as

$$p_m = p_{m\prime} \frac{\partial \overrightarrow{D_o}(x,y;g_y)/\partial y_m}{\partial \overrightarrow{D_o}(x,y;g_y)/\partial y_{m\prime}}, \quad [7a]$$

$$q_j = -p_{m\prime} \frac{\partial \overrightarrow{D_o}(x,y;g_y)/\partial b_j}{\partial \overrightarrow{D_o}(x,y;g_y)/\partial y_{m\prime}}. \quad [7b]$$

The resulting shadow price values represent the tradeoff values for producing more of the non-marketed desirable outputs and producing less of the undesirable outputs, in terms of foregone marketed output revenue. These are technology-based marginal trade-offs rather than welfare-based valuations.

Empirical Specification

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The translog and quadratic forms are most commonly used to parameterize distance functions. The translog functional form cannot satisfy the translation property of the directional distance function (Färe et al. 2008), it is typically employed for Shephard output distance functions. The quadratic form, as a second-order approximation to the true but unknown distance function, is suited to directional output distance functions (R. Chambers et al. 2013). Note that the quadratic specification of the directional distance function is designed to prioritize local accuracy at the extremes of the data support. Following Färe et al. (2005), we specify the directional output distance function empirically using the quadratic approximation(Aigner and Chu 1968), which is both parametric and twice differentiable. This specification allows us to derive the shadow price ratios above, as well as the output elasticity of substitution values discussed in Section 3.4. We specify the quadratic directional output distance function for each production unit, $k = 1, \ldots, K$, in time $t = 1, \ldots, T$, as

$$\begin{aligned} \vec{D}_o(x_k^t, y_k^t, b_k^t; \mathbf{g}) \quad &= \alpha + \textstyle\sum_n^N \alpha_n\, x_n + \sum_m^M \beta_m\, y_m + \sum_j^J \gamma_j\, b_j \\ &+ \tfrac{1}{2}\Big(\textstyle\sum_n^N \sum_{n'}^N \alpha_{nn'}\, x_n x_{n'} + \sum_m^M \sum_{m'}^M \beta_{mm'}\, y_m y_{m'} + \sum_j^J \sum_{j'}^J \gamma_{jj'}\, b_j b_{j'}\Big) \\ &+ \textstyle\sum_n^N \sum_m^M \delta_{nm}\, x_n y_m + \sum_n^N \sum_j^J \eta_{nj}\, x_n b_j + \sum_m^M \sum_j^J \mu_{mj}\, y_m b_j, \end{aligned} \qquad [8]$$

and

$$\alpha = \alpha_0 + \sum_k^K \phi_s\, CD_k + \sum_t^K \psi_t\, TD_t,$$

where CD_k and TD_t denote county and time fixed effects. We estimate the model parameters by solving a constrained deterministic linear programming (LP) problem (Aigner and Chu 1968). We choose a set of parameters $\left(\alpha_0, \alpha_n, \alpha_{nn}, \beta_m, \beta_{mm}, \gamma_j, \gamma_{jj}, \delta_{nm}, \eta_{nj}, \mu_{mj}, \phi_s, \psi_t\right)$ to minimize the sum of the distances over all production units and time, with the full formulation reported in

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Appendix B. The optimization problem explicitly imposes the standard axioms of production theory, including monotonicity, translation, and symmetry.

Direction Vector and Bias-correction

We explicitly specify the selection of the directional vector, $\mathbf{g} = (g_{ym}, g_{bj})$, in the DDF. Following the approach proposed by Färe et al. (2017), we endogenize the direction vector, allowing for the LP technique to optimize over the direction vector simultaneously. The optimized direction vector is common across all observations and periods, ensuring comparability of distance and shadow price estimates. Specifically, the direction vector is treated as coefficients to minimize the DDF. Prior to this, the vast majority of empirical studies (see in Färe et al., 2005; Färe et al., 2006; Fukuyam and Weber 2008; Fukuyama and Weber 2009; Bostian and Herlihy 2014; Yang et al. 2021) relied on a predetermined direction vector $(\mathbf{1}, -\mathbf{1})$ for simultaneously expanding the desirable outputs and contracting the undesirable outputs by the same degree. One advantage of choosing a unit direction vector is that it facilitates aggregation of inefficiency values, as well as standardizes their interpretation across observations. With respect to the outcomes of the selection, efficiency scores are often overstated (Fukuyama and Weber 2009; Murty et al. 2012), which imposes concerns on the credibility of results.

To facilitate comparison with the existing literature, we introduce alternative constraints on the direction vector:

$$g_{y,m} \geq 0 \text{ and } g_{b,j} \geq 0, \text{ for } m = 1, \ldots, M; b = 1, \ldots, J \quad [9a]$$

$$\sqrt{\sum_{m=1}^{M} (g_{y,m})^2 + \sum_{j=1}^{J} (g_{b,j})^2} = 1 \quad [9b]$$

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The non-negative constraint shown in equation [9a] restricts the expansion of desirable outputs and contraction of undesirable outputs to approaching the frontier. Equation [9b] restricts a unit length of the optimal directional vector, namely the norm of the direction vector is 1, to compare with any other arbitrary vector. Note that the arbitrary directional vector $(\mathbf{1}, -\mathbf{1})$ in this case is converted to $\left(\sqrt{\mathbf{1/5}}, -\sqrt{\mathbf{1/5}}\right)$ to satisfy the unit constraint.

We calculate shadow prices based on a feasible set of parameters $\widehat{\boldsymbol{\theta}} = \left(\alpha_0, \alpha_n, \alpha_{nn}, \beta_m, \beta_{mm}, \gamma_j, \gamma_{jj}, \delta_{nm}, \eta_{nj}, \mu_{mj}, \phi_s, \psi_t\right)$, which determines the frontier model. Since the true frontier is not observable and we only estimate it using a given sample, $\widehat{\boldsymbol{\theta}}$ ignores any stochastic factors which could bias the frontier and corresponding shadow price estimation. Therefore, we implement a bootstrap procedure (Efron 1979) to correct for any sampling bias of the estimates. Leopold Simar and Wilson (1998; Léopold Simar and Wilson 2000) introduce the procedure for for non-parametric efficiency estimates. Bostian and Herlihy (2014) adapt this procedure to parametric estimation, which we also follow here. The steps are:

(1) Estimate Appendix B, equation [15] for $\widehat{\boldsymbol{\theta}}$, and recover $\vec{D}_o$ for each unit k in each period t, denoted by $\widehat{\vec{D}_o}$; Calculate shadow price based on [7a] and [7b].

(2) Project each observation to the estimated frontier and account for stochastic influence by randomly assigning a draw, with replacement, from the distribution of $\widehat{\vec{D}_o}$, denoted by $\widehat{\vec{D}_o}^*$. A so-called pseudo sample has the output vector as $\left(y_b + g_y\left(\widehat{\vec{D}_o} - \widehat{\vec{D}_o}^*\right); b_b - g_b\left(\widehat{\vec{D}_o} - \widehat{\vec{D}_o}^*\right)\right)$, and the input vector remains unchanged.

(3) Estimate using pesudo sample for $\widehat{\boldsymbol{\theta}_b}$ and corresponding distance values.

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(4) Repeat steps (2)-(3) for $B = 2000$ times and restore the coefficients to derive standard error and calculate bias-corrected estimates such that:

$$bias = \frac{1}{B}\sum_b^B\left(\widehat{\boldsymbol{\theta}}_b - \widehat{\boldsymbol{\theta}}\right) \quad [10a]$$

$$\widehat{\boldsymbol{\theta}_{bias-corrected}} = \widehat{\boldsymbol{\theta}} - bias = 2 \times \widehat{\boldsymbol{\theta}} - \frac{1}{B}\sum_b^B \widehat{\boldsymbol{\theta}}_b \quad [10b]$$

$$se_{boot}\left(\widehat{\boldsymbol{\theta}}\right) = \sqrt{\frac{1}{B-1}\sum_b^B\left(\widehat{\boldsymbol{\theta}} - \widehat{\boldsymbol{\theta}}_b\right)^2} \quad [10c]$$

Morishima Elasticity of Substitution

The Morishima elasticity of substitution (MES) (Morishima 1967; Blackorby and Russell 1981) extends Hicksian elasticity of substitution concepts to production processes with multiple inputs and outputs. It measures the change in output ratios in response to relative changes in output prices, holding input constant. (Blackorby and Russell 1981). As with the shadow prices, we derive the MES from our quadratic specification of the directional output distance function in [8]. The corresponding MES for outputs are as follows, first between desirable and undesirable outputs,

$$M_{b_j y_m} = \frac{\partial ln(q_j/p)}{\partial ln(y/b_j)} = y_m^*\left(\frac{D_{b_j y_m}}{D_{b_j}} - \frac{D_{y_m y_m}}{D_{y_m}}\right). \quad [11]$$

then between undesirable outputs,

$$M_{b_j, b_{j\prime}} = \frac{\partial ln(q_j/q_{j\prime})}{\partial ln(b_{j\prime}/b_j)} = b_{j\prime}^*\left(\frac{D_{b_j b_{j\prime}}}{D_{b_j}} - \frac{D_{b_{j\prime} b_{j\prime}}}{D_{b_{j\prime}}}\right). \quad [12]$$

and finally, between desirable outputs,

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$$M_{y_m,y_{m'}} = \frac{\partial ln(p_m/p_{m'})}{\partial ln(y_{m'}/y_m)} = y^*_{m'}\left(\frac{D_{y_m,y_{m'}}}{D_{y_m}} - \frac{D_{y_{m'},y_{m'}}}{D_{y_{m'}}}\right). \quad [13]$$

where $y^*_m = y_m + \vec{D}_o(x, y, b; g)g_{y_m}$ and $b^*_j = b_j + \vec{D}_o(x, y, b; g)g_{b_j}$ represent the optimal values of outputs projected to the frontier in the g direction.

MES measures how the shadow-price ratio, $p_m/p_{m'}$, responds to changes in the output ratio, $y_{m'}/y_m$. In other words, MES reflects how sensitive the trade-off between outputs is at different points on the production frontier (Färe et al. 2005). A positive MES indicates that the shadow price ratio increases with the output ratio, implying that outputs m and m' are Morishima substitutes. Conversely, a negative MES suggests that the shadow price ratio decreases with the output ratio, indicating Morishima complementarity. Since MESs are inherently asymmetric ($M_{y_{m,m'}} \neq M_{y_{m',m}}$). They provide insight into the relative costs or difficulties of preserving one additional unit of a desirable output or reducing undesirable outputs (Färe et al. 2005, 2012; Kumar and Managi 2011; Bonilla et al. 2018). This asymmetry can further be decomposed into a proportional effect of $y_{m'}$ on variations in p_m, and a proportional effect of p_m on variation in $y_{m'}$ (Blackorby and Russell 1989).

However, it should be emphasized that the MES represents a local property of the estimated distance values. Since the elasticities are derived from the marginal trade-offs implied by the DDF, they characterize substitution patterns around the observed production bundles and along the estimated frontier. Consequently, the MES values should be interpreted as describing local substitution relationships rather than global substitution possibilities across the entire production technology. Besides, the magnitude of the elasticities depends on the assumed functional form of the production technology.

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4. Data and Empirical Results

Data

The boreal forest production technology consists of multiple inputs and outputs using county-by-year panel data available from 2008 to 2014. The forests across 21 counties are actively managed to produce biomass of different uses including GROT (W. Zhou et al., 2019; 2014), which contributes to bioenergy. We define a forestry production process with two inputs, four desirable outputs, and one undesirable output. Inputs include the standing volume of forests, measured in million m^3, and labor measured by annual working units (AWU). The desirable outputs consist of marketed goods and ecosystem services. Sawtimber, pulpwood, and fuelwood are harvested and collected on-site and measured in m^3 solid volume under bark. We use a weighted sum of sawtimber and pulpwood to represent timber production since they are highly correlated and the two types of biomass can be obtained from one single tree with a relatively fixed proportion. For instance, the data demonstrate that, on average, 47.3% of the harvest becomes sawtimber, 44.4% is processed to pulpwood and the residuals (8.3%) are used for fuelwood. We use the weighted sum to describe timber production and treat it as one output in the model specification. We calculate a weighted average price for the weighted timber output, including sawtimber and pulpwood price[iii] (data source: online database of Swedish Forest Agency).

Bioenergy in our framework is a composite marketed output derived from multiple biomass streams. It consists of GROT, measured in 1,000 tons of dry matter, as well as fuelwood and the share of sawtimber and pulpwood that is ultimately used for energy production. GROT is a harvesting residue collected on site; in accordance with Swedish Forestry Agency regulations (Backlund 2008), at least 20% must remain on the harvest site, and only the extracted portion is

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included in our measure. The bioenergy price is constructed as a weighted average of the relevant wood components contributing to energy production, using observed market prices (SEK/MWh) for wood chips delivered to district heating. As such, bioenergy reflects an integrated revenue stream within the forest production system rather than a single externally priced commodity.

The technology jointly produces environmental goods or ecosystem services. This includes biodiversity, here proxied by the volume of deadwood left in the forests (in million m^3 over bark), and carbon sequestration accounted for by carbon generated and stored in the annual growth of the forests.

Biodiversity is proxied by deadwood volume, a widely used structural habitat indicator in boreal forest ecology due to its strong association with saproxylic species and habitat complexity (Lassauce et al. 2011). We do not differentiate between sources of deadwood (e.g., natural mortality, harvest residues, or regulatory set-asides), as these components are aggregated in the available data. While informative, this measure captures only one structural dimension of biodiversity. Accordingly, the estimated shadow prices should be interpreted as marginal opportunity costs of increasing deadwood-defined biodiversity, rather than as valuations of biodiversity more broadly conceived.

There are two sources of carbon emissions. When burning or combusting GROT, the technology instantaneously emits CO_2, measured by 1000 tons. Another source of carbon emission is part of timber production considered by-products that are combusted, i.e., approximately 38% of sawtimber becomes byproducts and emits carbon, and 55% of pulpwood is combusted and emits carbon. The share of sawtimber and pulpwood by-products used for energy is based on data from

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the Forest Industry's IT company (SDC) (W. Zhou et al. 2019). We assume that undesirable non-marketed outputs are weakly disposable, implying that the producer will face a cost to reduce the level of carbon emissions. The marketed and non-marketed desirable outputs are assumed to be freely disposable, meaning they can be reduced without costs. A detailed input-output description is presented in Table 1.

<<Table 1 about here>>

Estimation and Results

Parameter estimations are reported in Table 2 and is estimated by utilizing the SLSQP (Sequential Least SQuares Programming), an iterative method for constrained non-linear optimization, minimizing scalar functions with equality/inequality constraints and bounds(Kraft 1988). solver in SciPy. We assess monotonicity[iv], non-jointess[v] and concavity[vi] in Table A5 in the Appendix. We also assess the impact of choosing the conventional directional vector, $\left(\sqrt{\mathbf{1/5}}, -\sqrt{\mathbf{1/5}}\right)$ (Färe et al. 2017), as another robustness check. We show that both specifications satisfy these properties. Note that we normalize input and output variables by dividing by their mean value to overcome convergence problems (Färe et al. 2005), thus shadow prices should be scaled by a mean-corrected ratio:

<<Table 2 about here>>

$$p_m = p_{m\prime} \frac{\partial \overrightarrow{D_o}(x,y;g_y)/\partial y_m}{\partial \overrightarrow{D_o}(x,y;g_y)/\partial y_{m\prime}} \times \frac{\overline{y_{m\prime}}}{\overline{y_m}}, \quad [14a]$$

$$q_j = -p_{m\prime} \frac{\partial \overrightarrow{D_o}(x,y;g_y)/\partial b_j}{\partial \overrightarrow{D_o}(x,y;g_y)/\partial y_{m\prime}} \times \frac{\overline{y_{m\prime}}}{\overline{b_j}}, \quad [14b]$$

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where $\overline{y_{m'}}$ denotes the mean of the referenced output, $\overline{y_m}$ and $\overline{b_j}$ denote the mean of non-marketed desirable and undesirable outputs, respectively. We select bioenergy as the reference output because it represents the most comprehensive marketed transformation of harvested biomass within the production system, linking timber assortments and residual streams. Bioenergy is constructed as a composite marketed output, with its price defined as a weighted average of the relevant wood components contributing to energy production. As such, it reflects the integrated revenue structure of forest biomass utilization rather than a single exogenous market price. We also conduct the bootstrap procedure to account for stochastic influence, and report the bias-corrected estimates associated with bootstrapped standard errors. In addition, the bootstrap procedure provides a robust basis for inference, particularly when the underlying distributions are unknown for the estimates stemming from linear programming. We summarize these bias-corrected estimates with the associated standard errors, and we show that bias-corrected estimates are not significantly different from the original estimates. Standard errors also indicate the the 5% of significant level of the estimates. Spatial and temporal effects are captured by dummy variables and the estimates are presented in the Appendix; the conclusion is that they are significant throughout the bootstrap procedure. the values in $(g_{y,1}, g_{y,2}, g_{y,3}, g_{y,4}, g_{b,1})$ provide information for the directional vector selected by the algorithm, which intends to minimize the sum of distances across each county over the whole period.

Distance and Shadow Price

Table 3 summarizes statistics of the distance function and shadow prices. There are 38 observations located on the frontier with a distance value of zero. A mean distance value or

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inefficiency is 0.01. A high efficiency score may be due to the fact that we involve the directional vector explicitly into the optimization, and such a vector value indicates the best possible path toward the frontier. To interpret, distance values represent potential efficiency improvement and should be re-scaled by the mean value of outputs and the directional vector. For instance, the average output expansion of carbon sequestration (y_4) is $0.01 \times 1539.72 \times 1000 \times 0.0715 = 1100.90$ ton through the optimal efficiency improvement path for a representative county. Deadwood expansion (y_3) is $0.01 \times 8.31 \times 1000000 \times 0.0264 = 2139.84\ m^3$ for a county operating on the efficient frontier. Simultaneously, carbon emission (b_1) can be potentially reduced by $0.01 \times 385.12 \times 1000 \times 0.7127 = 2744.75$ ton through efficiency improvement.

<<Table 3 about here>>

In Figure 1 (a), we depict full sample efficiency evolution and regional efficiency[vii] over time. There is an increasing trend of inefficiency over time. Götaland region demonstrates inefficiency growth surpassing the mean value of the full sample inefficiency.

<<Figure 1 about here>>

The shadow price of a desirable output represents the implicit or opportunity cost of increasing one unit of that output. For instance, the shadow price of biodiversity reflects the cost, in terms of foregone revenue, that the forest production process incurs to increase biodiversity by an additional unit (1 cubic meter of dead wood). Our estimates of the opportunity cost for biodiversity, proxied by the volume of deadwood is around SEK 3 per m^3, with bioenergy production as a reference output, which means to increase deadwood by 1 m^3, the total revenue must decrease by 3 SEK for a county operating on the frontier. Similarly, that of carbon

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sequestration is estimated at SEK 50 per ton. To keep an additional ton of sequestered carbon, the total revenue must decrease by 50 SEK. The shadow price for the undesirable output can be viewed as the marginal abatement cost, which is informative for environmental policies such as emission trading permits and taxation. We show that the abatement cost of carbon emission is about SEK 2000 per ton, that is, to reduce carbon emission by 1 ton, there will be a 2000 SEK revenue loss in total.

Figure 1 (b) - (d) demonstrates the temporal changes in shadow prices. There is no clear trend for the three non-marketed outputs, as they fluctuated over the period and they peaked in 2011. For the study period, the shadow price for biodiversity declines by 5.4%, and that for carbon sequestration increases by 3.8%. The shadow price, or implied abatement cost, for carbon emission drops by 4.1%.

In Figure 2 (a)-(c), we visualize the relationship between (average) inefficiency and opportunity cost (or abatement cost for emissions) for each pair of non-marketed outputs by county. The size of the bubble indicates the level of inefficiency; larger size means higher inefficiency. We observe a distinct clustering effect in the color categories, particularly among counties in the Svealand and Götaland regions, which are located in southern Sweden. These regions host cities that serve as hubs for the most active economic activities, making it plausible that they exhibit higher opportunity costs. Beyond eyeballing the observed clustering, we employ a two-way fixed-effects model to analyze pairwise relationships among these ESs. The results, reported in Appendix Table A7, indicate that the shadow price of carbon sequestration exhibits negative correlations with biodiversity and carbon emissions individually. This suggests that regions where enhancing one ecosystem service is costly tend to face relatively low opportunity costs for

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the other, highlighting spatial trade-offs that are crucial for designing cost-effective forest policies.

<<Figure 2 about here>>

Figure 3 shows the geographical distribution of average shadow prices for biodiversity, carbon sequestration, and carbon emission across the counties. The counties surrounding Stockholm and those in the southern region exhibit higher shadow prices for biodiversity. This implies that increasing biodiversity by one unit in these areas results in greater foregone forest production values compared to northern counties. Conversely, shadow prices for carbon sequestration are distributed in northern regions rather than in the Stockholm area, aligning with the previously described negative correlation between biodiversity and carbon sequestration. Shadow prices for carbon emissions are concentrated in both northern and southern counties. It should be noted that the range of carbon emission shadow prices across all counties are relatively narrow, varying only by approximately 10% from a low of SEK 1840 to a high of SEK 2060 per ton. This indicates a high degree of similarity in emission shadow prices among all counties.

<<Figure 3 about here>>

The observed spatial variation in shadow prices suggests that uniform national compensation schemes may not be cost-effective. Conditional on a given environmental target, spatially differentiated payments for ecosystem services (PES) could reduce aggregate opportunity costs by prioritizing counties where marginal trade-offs are lower.

Output substitution: Morishima Elasticities

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To facilitate interpretation, we focus the discussion on a subset of Morishima elasticities that capture the main trade-offs among forest outputs in Table 4. The full set of elasticities is reported in Appendix Table A8. Many of the remaining elasticities are numerically close to zero and therefore provide limited additional information for interpreting substitution patterns. Note that these elasticities represent local substitution measures implied by the estimated DDF and therefore characterize substitution patterns around observed production points.

<<Table 4 about here>>

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The elasticities between timber and bioenergy indicate a substitution relationship between the two outputs. The positive elasticity from timber to bioenergy (1.35) suggests that an increase in the relative production of bioenergy raises the shadow price ratio of timber to bioenergy, implying increased scarcity of timber. Conversely, the negative elasticity from bioenergy to timber (-1.67) indicates that expanding timber production tends to reduce the relative cost of bioenergy.

From a policy perspective, the substitution relationship suggests that policies promoting forest-based bioenergy may increase competition for forest timber biomass. Higher demand for bioenergy or subsidies for bioenergy can reallocate forest resources toward energy production, increasing the opportunity cost of supplying timber. The negative elasticity from bioenergy to timber suggests that increasing timber production reduces the relative scarcity of bioenergy. In other words, expanding timber output indirectly facilitates bioenergy supply by increasing the availability of forest biomass. These elasticities imply that expanding bioenergy production intensifies competition for forest biomass and therefore raises the opportunity cost of timber. Conversely, greater timber production may also benefit bioenergy supply by increasing the overall stock of available biomass.

The elasticities between biodiversity and carbon sequestration are both positive, suggesting a trade-off between these two ecosystem services. Increasing carbon sequestration raises the relative scarcity of biodiversity, indicating that carbon-focused forest management may increase the opportunity cost of biodiversity conservation. Conversely, biodiversity-oriented management may also constrain carbon sequestration potential. The asymmetric magnitudes in the elasticities suggest that carbon-oriented policies, such as climate policies, may exert stronger effects on biodiversity outcomes than the reverse.

The elasticity from timber to carbon sequestration is -12.48, which is substantially larger in magnitude than the reverse elasticity from carbon sequestration to timber (-2.49). This pattern reflects the competing use of forest resources for timber production versus carbon storage. Specifically, increasing carbon sequestration substantially lowers the opportunity cost of timber production, while expanding timber harvesting more modestly reduces the relative scarcity of carbon sequestration. Therefore, policies aimed at expanding either output will affect the relative scarcity of the other. Our estimated elasticities provide quantitative evidence of this asymmetric trade-off: policies prioritize timber supply tend to affect climate objectives more intensively than the reverse.

We observe similar patterns between timber production and biodiversity as in both Morishima elasticities are negative, indicating a complementary relationship between the two outputs. Again, we stress that biodiversity in this study is proxied by the volume of deadwood, which includes sources of natural felling, harvesting residues and regulatory requirements. While this measure may not fully reflect the ecological complexity of biodiversity, these elasticities still provide guidance for policy design between biodiversity management and timber production. Overall, the MES framework provides quantitative evidence for policymakers for balancing

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competing forest management objectives, especially insights into trade-offs among ecosystem services and production goals.

5. Conclusion

This study develops a comprehensive production technology for Swedish boreal forests that jointly accounts for market outputs, non-market ecosystem services, and carbon emissions within a multiple-output directional distance framework. By estimating shadow prices for both desirable and undesirable outputs and examining substitution patterns through Morishima elasticities, we shed new light on the trade-offs linking timber and bioenergy production with biodiversity conservation and carbon sequestration. Methodologically, the analysis extends existing approaches by endogenizing the directional vector, thereby refining the measurement of shadow prices.

By quantifying shadow prices, we make explicit the marginal opportunity costs of enhancing biodiversity or carbon sequestration within the existing forest production technology. These estimates reflect production-side marginal rates of transformation expressed in terms of foregone marketed revenue. They do not represent social willingness to pay, welfare-based valuations, or socially optimal provision levels. Rather, they capture the cost dimension of ecosystem service supply conditional on the observed technology and market prices.

These estimates can be interpreted as providing a supply-side marginal cost for ecosystem service provision across regions. Conditional on an exogenously determined environmental objective, these shadow price estimates can inform least-cost policy design. For example, if policymakers aim to increase national deadwood volumes (related to biodiversity) by a specified amount, the estimated opportunity costs provide a benchmark for the minimal compensation

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required to induce participation from private forest owners. Moreover, the documented spatial heterogeneity in shadow prices suggests that conservation incentives could be achieved geographically, for instance, a reduced aggregate cost for a given biodiversity target in the northern counties. In this sense, our results inform the implementation and spatial targeting of environmental policies, without prescribing the socially optimal level of ecosystem service provision.

While these findings provide valuable policy insights, some limitations are also noted. Our estimates are derived from county-level averages and therefore represent aggregate marginal trade-offs within each region. In practice, forest management decisions occur at the stand or ownership level, where opportunity costs may vary substantially within counties. As a result, the reported shadow prices should be interpreted as regional cost benchmarks rather than precise compensation values for individual forest owners. Policies designed at finer spatial scales may require additional information to account for within-county heterogeneity. In addition, the framework does not incorporate feedback effects between atmospheric carbon concentrations and sequestration capacity (Hoel and Sletten 2016), which are likely to influence ESs over longer horizons. Future research could address these challenges by adopting more dynamic and spatially explicit approaches, such as network DEA (W. Zhou et al. 2019), that are suited to capturing intertemporal effects in forest ES provision. Nonetheless, our production framework provides a transparent and flexible basis for evaluating trade-offs among forest ecosystem services, which is crucial for designing policies where multiple objectives coexist in the absence of market prices.

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Table 1: Descriptive Statistics of Inputs and Outputs for Swedish Forest at County Level, 2008–2014

Variable Name	Variable	Explanation
Input		
Standing Volume	x_1	Standing volume on productive forest land
Annual Working Units (AWU)	x_2	Annual working units in forestry, which is the total number of working hours divided by 1800 hours
Desirable Output		

Variable Name	Variable	Explanation
Weighted sum of Timber assortments	y_1	Weighted sum of sawtimber and pulpwood
Sawtimber		Volume of sawtimber of annual felling
proportion to become bioenergy		proportion of sawtimber that will be used as bioenergy
Pulpwood		Volume of pulpwood of annual felling
proportion to become bioenergy		proportion of pulpwood that will be used as bioenergy
Bioenergy	y_2	
GROT		Amount of Grot (Branches and tops) extracted from harvested forest site
Fuelwood		Volume of fuelwood of annual felling
Sawtimber to bioenergy		Proportion of Sawtimber becomes bioenergy
Pulpwood to bioenergy		Proportion of Pulpwood becomes bioenergy
Biodiversity	y_3	Volume of deadwood left in forest
Carbon Sequestration	y_4	Carbon sinks in the annual growth
Undesirable Output		
Carbon Emission	b_1	Total carbon emission from combustion of bioenergy
Output Prices		

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Variable Name	Variable	Explanation
Timber Price	p_1	Weighted average price of delivered sawtimber and pulpwood
Bioenergy Price	p_2	Weighted average price of delivered byproducts and GROT

Panel (a): Input and Output Variable Explanation

Table 1 (continued)

Variable Name	Variable	Unit	N	Mean	St.D	Min	Max
Standing Volume	x_1	million m^3	147	143.98	94.95	13.70	332.70
Annual Working Units	x_2	1AWU=1800 h	147	747.65	413.77	91.00	2303.00
Weighted Sum of Timber assortments	y_1	1000 m^3	147	851.95	458.97	42.81	1889.37
Sawtimber		1000 m^3	147	1592.31	864.35	79.56	3661.89
proportion to become bioenergy		100%	147	0.38	0.0082	0.37	0.40
Pulpwood		1000 m^3	147	1470.09	794.15	73.96	2941.36
proportion to become bioenergy		100%	147	0.56	0.0049	0.55	0.57
Bioenergy	y_2	1000 m^3	147	1779.08	941.81	96.36	3695.41

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Variable Name	Variable	Unit	N	Mean	St.D	Min	Max
GROT		1000 tons	147	66.80	40.66	14.356	193.02
Fuelwood		1000 m^3	147	283.57	153.16	14.36	582.35
Sawtimber to bioenergy		1000 m^3	147	608.79	334.08	30.15	1421.71
Pulpwood to bioenergy		1000 m^3	147	819.92	443.09	41.04	1670.19
Biodiversity	y_3	million m^3	147	8.31	7.50	0.50	27.70
Carbon Sequestration	y_4	1000 tons	147	1539.72	952.24	111.72	3301.33
Carbon Emission	b_1	1000 tons	147	385.12	198.42	22.90	766.53
Timber Prcie	p_1	SEK per m^3	147	420.47	61.38	327.23	552.32
Bioenergy Price	p_2	SEK per ton	147	418.95	61.14	322.29	563.77

Panel (b): Descriptive Statistics of Inputs, Outputs, Undesirable Outputs, and Prices

Note: All output prices are expressed in constant 2015 SEK.

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Table 2: Parameter Estimation, Bias-corrected Estimation of DDF

Parameter	Variable	Estimate	Bias-corrected Estimate	Standard Error
α_0	Constant	-0.0055	-0.0051	0.0016
α_1	x_1	0.1563	0.1501	0.0132
α_2	x_2	0.0209	0.0206	0.0054
β_1	y_1	0.0003	0.0002	0.0000
β_2	y_2	-0.7189	-0.7180	0.0009
β_3	y_3	-0.0436	-0.0443	0.0010
β_4	y_4	0.0129	0.0129	0.0001

Parameter	Variable	Estimate	Bias-corrected Estimate	Standard Error
γ_1	b_1	0.6970	0.6968	0.0005
α_{11}	x_1^2	-0.2708	-0.2675	0.0082
α_{22}	x_2^2	0.0557	0.0571	0.0044
β_{11}	y_1^2	-0.0037	-0.0039	0.0006
β_{22}	y_2^2	-0.0001	-0.0001	0.0000
β_{33}	y_3^2	-0.0000	-0.0000	0.0000
β_{44}	y_4^2	0.0000	0.0000	0.0000
γ_{11}	b_1^2	0.0000	0.0000	0.0000
δ_{11}	x_1y_1	-0.0208	-0.0201	0.0011
δ_{12}	x_1y_2	0.0166	0.0175	0.0006
δ_{13}	x_1y_3	0.0326	0.0325	0.0007
δ_{14}	x_1y_4	0.0820	0.0809	0.0024
δ_{21}	x_2y_1	0.0011	0.0011	0.0003
δ_{22}	x_2y_2	-0.0000	-0.0018	0.0015
δ_{23}	x_2y_3	0.0078	0.0081	0.0003
δ_{24}	x_2y_4	-0.1682	-0.1672	0.0015
η_{11}	x_1b_1	0.0226	0.0221	0.0010
η_{21}	x_2b_1	-0.0107	-0.0099	0.0012
μ_{11}	y_1b_1	0.0000	0.0000	0.0000

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Parameter	Variable	Estimate	Bias-corrected Estimate	Standard Error
μ_{21}	$y_2 b_1$	0.0000	0.0000	0.0000
μ_{31}	$y_3 b_1$	0.0000	0.0000	0.0000
μ_{41}	$y_4 b_1$	0.0000	0.0000	0.0000
α_{12}	$x_1 x_2$	0.1433	0.1421	0.0013
β_{12}	$y_1 y_2$	-0.0015	-0.0011	0.0004
β_{13}	$y_1 y_3$	-0.0006	-0.0008	0.0006
β_{14}	$y_1 y_4$	0.0222	0.0216	0.0008
β_{23}	$y_2 y_3$	0.0014	0.0010	0.0004
β_{24}	$y_2 y_4$	0.0007	0.0006	0.0001
β_{34}	$y_3 y_4$	-0.0210	-0.0202	0.0014
$g_{y,1}$		0.0000	0.0001	0.0008
$g_{y,2}$		0.6980	0.6986	0.0014
$g_{y,3}$		0.0236	0.0220	0.0031
$g_{y,4}$		0.0466	0.0363	0.0099
$g_{b,1}$		0.7143	0.7145	0.0014
$\boldsymbol{\phi_s}$	County dummy	–		
$\boldsymbol{\psi_t}$	Year dummy	–		

Note: County and Year dummies are reported in Appendix A, Table A4.

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Table 3: Results under Optimal Directional Vector

Non-marketed Outputs	N	Mean	St. Dev	Min	Max
Bias-corrected					
Distance Value	147	0.010	0.012	0.000	0.061
Shadow price					
Deadwood (y_3)	147	2.744	1.492	0.000	6.195
Carbon Sequestration (y_4)	147	49.222	51.932	0.000	270.349
Carbon Emission (b_1)	147	1955.825	275.251	1484.587	2561.246

Table 4: Selected Morishima Elasticities between Forest Outputs

Elasticity	Mean(SD)	Reverse elasticity	Mean(SD)
Timber → Bioenergy	1.3458	Bioenergy → Timber	-1.6698
	(1.1314)		(1.3985)
Timber → Carbon Sequestra-tion	-12.4789	Carbon Sequestration → Timber	-2.4865
	(11.3128)		(1.8572)
Biodiversity → Carbon Sequestration	3.2262	Carbon Sequestration → Biodiversity	0.6021
	(5.4628)		(0.8403)
Biodiversity → Timber	-1.7515	Timber → Biodiversity	-0.2086
	(1.3814)		(0.1853)

Notes: This table reports a selected set of Morishima elasticities that are most relevant for the main forest management trade-offs involving timber, bioenergy, biodiversity, and carbon sequestration. Entries report sample means, with standard deviations (SD) in the adjacent

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column. Morishima elasticities are asymmetric, so the elasticity from output i to output j generally differs from that from output j to output i. As implied by the directional distance function, these elasticities should be interpreted as local substitution measures evaluated around the observed production bundles on the estimated frontier.

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Figure Captions

Figure 1: Annual average distance and shadow prices by region

(a) Distance value

(b) Biodiversity (deadwood) shadow price

(c) Carbon sequestration shadow price

(d) Carbon emission shadow price

Figure 2: Relationship between annual average shadow prices by region

(a) Shadow price, biodiversity vs. carbon sequestration

(b) Shadow price, biodiversity vs. carbon emission

(c) Shadow price, carbon sequestration vs. carbon emission

Note: Bubble size denotes the level of inefficiency; larger bubbles mean higher inefficiency.

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Figure 3: Geographical heatmap of shadow prices by county, average value, 2008–2014

(a) Market price for timber (SEK/m^3)

(b) Shadow price for biodiversity (SEK/m^3)

(c) Shadow price for carbon sequestration (SEK/ton)

(d) Shadow price for carbon emission (SEK/ton)

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[i] Statistics and information provided by Swedish Forestry Industries.

[ii] Policies and path evolve in accordance to resolve challenges, the current framework is issued in Sep 2022. see: New EU forest strategy for 2030 — Sustainable Forest Management in Europe.

[iii] All prices and costs are in 2015 SEK, using alternative base years (2008) produces nearly identical estimates.

[iv] Monotonicity refers to the sign of the first-order derivatives from the distance function with respect to input and output variables. These derivatives are calculated at a mean levels. For instance, derivatives with respect to inputs and undesirable outputs should be non-negative and non-positive with respect to desirable outputs.

[v] Null-jointness means that zero undesirable output cannot be achieved while producing positive desirable output. Hence, $(x, y, 0)$ lies outside the technology set and the directional distance is non-positive, $\vec{D}_o(x, y, 0; g) \leq 0$, indicating infeasibility.

[vi] Concavity refers to assessing negative semi-definiteness of the Hessian matrix.

[vii] Four regions: Svealand, Götaland, Södra Norrland and Norra Norrland. Svealand consists of Stockholm, Uppsala, Södermanland, Värmland, Örebro, Västmanland, Dalarna. Götaland consists of Östergötland, Jönköping, Kronoberg, Kalmar, Blekinge, Skåne, Halland, Västra Götaland and Gotland. Södra Norrland consists of Gävleborg, Jämtland and Västernorrland. Norra Norrland consists of Västerbotten and Norrbotten.

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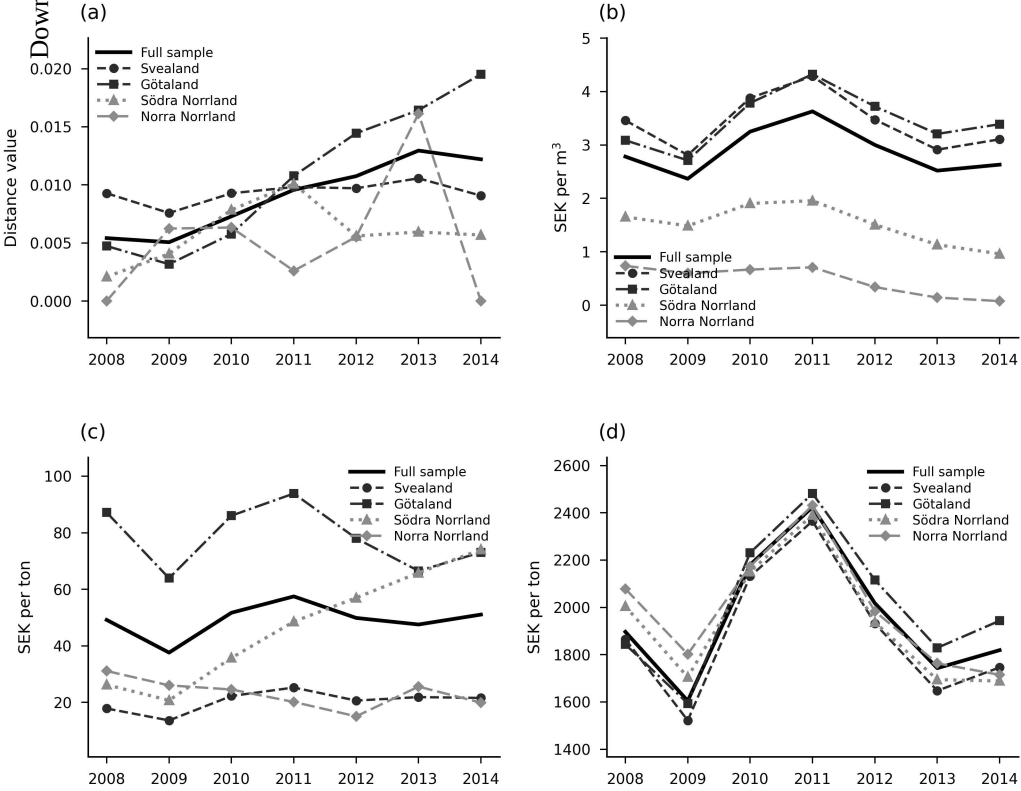
(a)
Distance value
0.020
0.015
0.010
0.005
0.000
Full sample
Svealand
Götaland
Södra Norrland
Norra Norrland
2008
2009
2010
2011
2012
2013
2014
(b)
SEK per m³
5
4
3
2
1
0
Full sample
Svealand
Götaland
Södra Norrland
Norra Norrland
2008
2009
2010
2011
2012
2013
2014
(c)
SEK per ton
100
80
60
40
20
Full sample
Svealand
Götaland
Södra Norrland
Norra Norrland
2008
2009
2010
2011
2012
2013
2014
(d)
SEK per ton
2600
2400
2200
2000
1800
1600
1400
Full sample
Svealand
Götaland
Södra Norrland
Norra Norrland
2008
2009
2010
2011
2012
2013
2014

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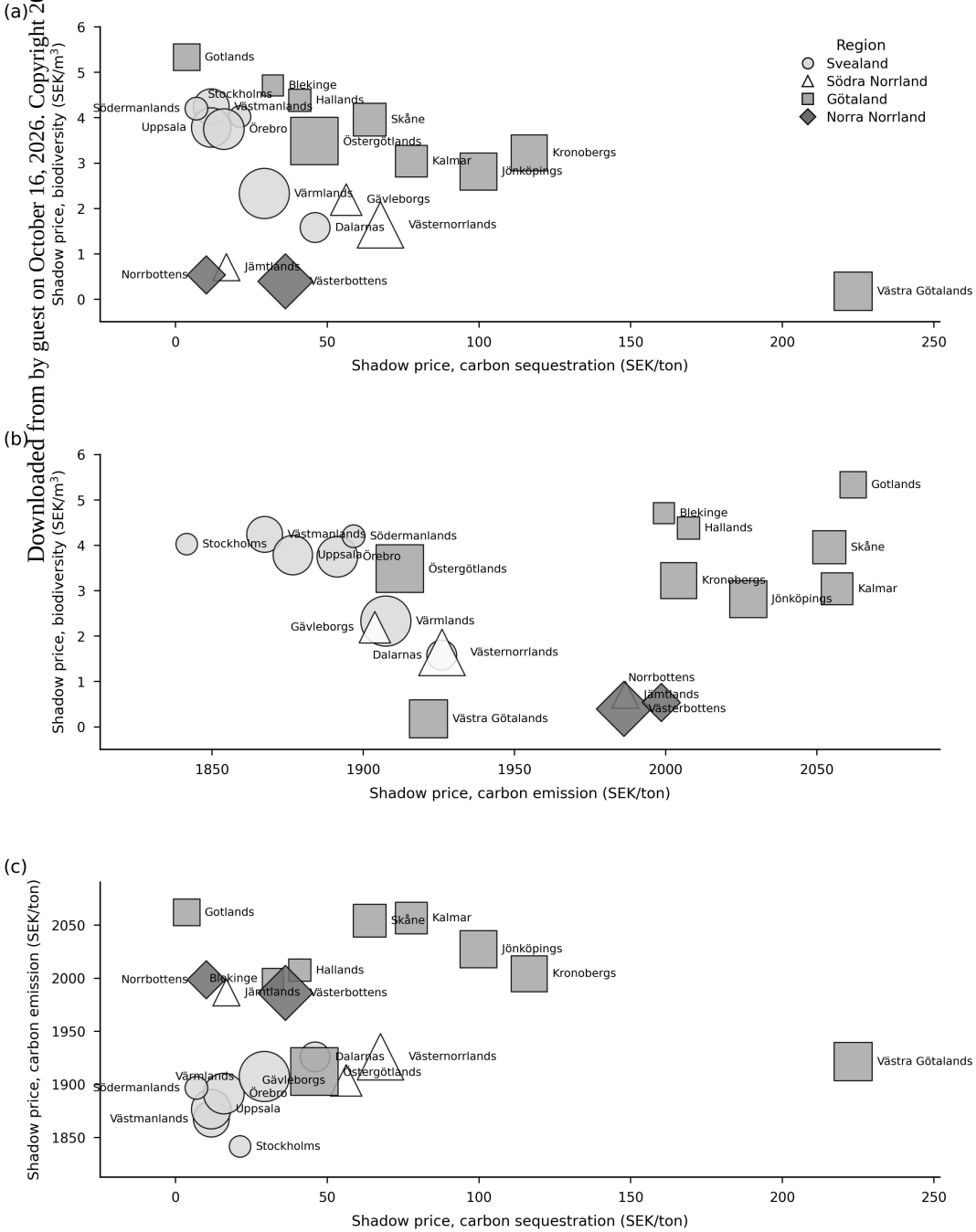
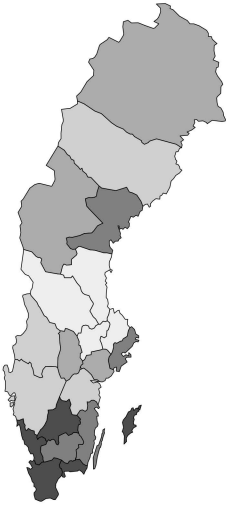
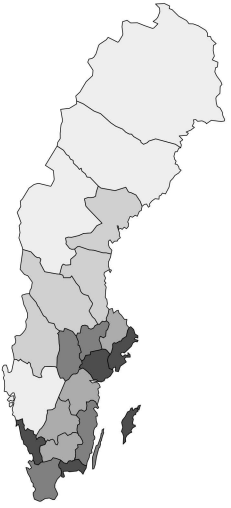




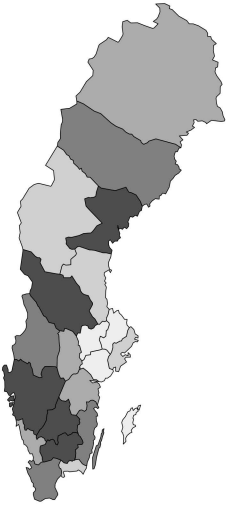
(a)

SEK/m^3
- 404–407
- 407–413
- 413–420
- 420–435
- 435–449



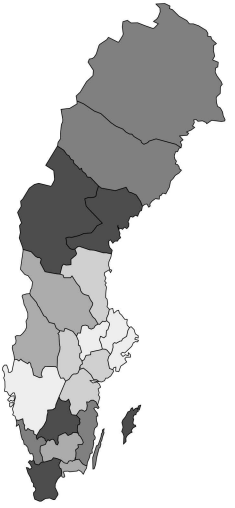
(b)

SEK/m^3
- 0.87–1.62
- 1.62–2.65
- 2.65–3.28
- 3.28–3.96
- 3.96–5.33



(c)

SEK/ton
- 3.65–24.7
- 24.7–35.4
- 35.4–53.2
- 53.2–67.6
- 67.6–112



(d)

SEK/ton
- 1,888–1,913
- 1,913–1,923
- 1,923–1,978
- 1,978–2,000
- 2,000–2,062