A6. Derivation of the relative magnitudes in Table 2

Let us assume $n \geq 2$. The conditions that define the Pareto optimal allocation in [8] can be written as follows:

$$\frac{U_3^*}{U_1^*} = \frac{1}{p_1 n}$$
$$\frac{U_2^*}{U_1^*} = \frac{p_2}{p_1}.$$

The conditions that define the allocation consistent with the optimal dedicated tax in [A8] and [12] can be written in parallel fashion as

$$\frac{U_3^N}{U_1^N} = \frac{1}{p_1\left(n + (n-1)\varepsilon_{x_2^N,t_2}\right)}$$
$$\frac{U_2^N}{U_1^N} = \frac{p_2 + t_2\left(1 - \frac{1}{n + (n-1)\varepsilon_{x_2^N,t_2}}\right)}{p_1}$$

where the second comes from substituting the first into [12] and rearranging. Comparing these conditions, it follows that

$$\frac{U_3^N}{U_1^N} \gtreqless \frac{U_3^*}{U_1^*} \Leftrightarrow \varepsilon_{x_2^N,t_2} \lesseqgtr 0$$

and

$$\frac{U_2^N}{U_1^N} \gtreqless \frac{U_2^*}{U_1^*} \Leftrightarrow \varepsilon_{x_2^N,t_2} \gtreqless -1.$$

Three cases are then useful to consider:

$$\varepsilon_{x_2^N,t_2} \geq 0 \Rightarrow \frac{U_3^*}{U_1^*} \geq \frac{U_3^N}{U_1^N} \text{ and } \frac{U_2^*}{U_1^*} < \frac{U_2^N}{U_1^N}$$

$$\varepsilon_{x_2^N,t_2} \in (-1,0) \Rightarrow \frac{U_3^*}{U_1^*} < \frac{U_3^N}{U_1^N} \text{ and } \frac{U_2^*}{U_1^*} < \frac{U_2^N}{U_1^N}.$$

$$\varepsilon_{x_2^N,t_2} \leq -1 \Rightarrow \frac{U_3^*}{U_1^*} < \frac{U_3^N}{U_1^N} \text{ and } \frac{U_2^*}{U_1^*} \geq \frac{U_2^N}{U_1^N}.$$

The results in Table 2 follow by aligning each of these conditions with strict concavity with respect to each argument of the utility function, along with use of the budget constraint.