

## Appendix D: Elasticity Formula Derivations

### Elasticity Equation

$$\widehat{elast}_i = \frac{\partial P_{it}}{\partial WQ_{it}} \frac{WQ_{it}}{P_{it}} \quad [D1]$$

### Log-log Model Specification

$$\begin{aligned} \ln(P_{it}) = & \beta_0 + \beta_1 \ln(WQ_{it}) + \beta_2 \ln(WQ_{it}) * LF_i + \beta_3 \ln(WQ_{it}) * \ln(Dist_i) \\ & + \beta_4 \ln(WQ_{it}) * \ln(Area_i - \overline{Area}) + \beta_5 \ln(WQ_{it}) * U_i + \beta_E \ln(WQ_{it}) * E_i \\ & + \beta_S \ln(WQ_{it}) * S_i + \beta_{Prop} Prop_i + \beta_{Control} Control_i + \gamma_{it} + \tau_{it} + \epsilon_{it} \end{aligned} \quad [D2]$$

### Talking partial derivative

$$\begin{aligned} \frac{\partial P_{it}}{\partial WQ_{it}} = \\ P_{it}/WQ_{it} * (\hat{\beta}_1 + \hat{\beta}_2 * LF_i + \hat{\beta}_3 \ln(Dist_i) + \hat{\beta}_4 \ln(Area_i - \overline{Area}) + \hat{\beta}_5 U_i + \hat{\beta}_E E_i + \hat{\beta}_S S_i) \end{aligned} \quad [D3]$$

### Rearranging

$$\begin{aligned} \hat{\beta}_1 + \hat{\beta}_2 * LF_i + \hat{\beta}_3 \ln(Dist_i) + \hat{\beta}_4 \ln(Area_i - \overline{Area}) + \hat{\beta}_5 U_i + \hat{\beta}_E E_i + \hat{\beta}_S S_i = \\ \frac{\partial P_{it}}{\partial WQ_{it}} \frac{WQ_{it}}{P_{it}} \end{aligned} \quad [D4]$$

### Thus

$$\frac{\partial \ln(P_{it})}{\partial \ln(WQ_{it})} = elast_i \quad [D5]$$

### Log-linear Model Specification

$$\begin{aligned} \ln(P_{it}) = & \beta_0 + \beta_1 WQ_{it} + \beta_2 WQ_{it} * LF_i + \beta_3 WQ_{it} * \ln(Dist_i) \\ & + \beta_4 WQ_{it} * \ln(Area_i - \overline{Area}) + \beta_{Prop} Prop_i + \beta_{Control} Control_i \\ & + \gamma_t + \tau_{it} + \epsilon_{it} \end{aligned} \quad (D6)$$

Taking partial derivative

$$\begin{aligned} \frac{\partial P_{it}}{\partial WQ_{it}} = \\ P * (\hat{\beta}_1 + \hat{\beta}_2 * LF_i + \hat{\beta}_3 \ln(Dist_i) + \hat{\beta}_4 \ln(Area_i - \overline{Area}) + \hat{\beta}_5 U_i + \hat{\beta}_E E_i + \hat{\beta}_S S_i) \end{aligned} \quad (D7)$$

Rearranging and multiplying by  $WQ_{it}$

$$\begin{aligned} WQ_{it} * (\hat{\beta}_1 + \hat{\beta}_2 * LF_i + \hat{\beta}_3 \ln(Dist_i) + \hat{\beta}_4 \ln(Area_i - \overline{Area}) + \hat{\beta}_5 U_i + \hat{\beta}_E E_i + \hat{\beta}_S S_i) = \\ \frac{\partial P_{it}}{\partial WQ_{it}} \frac{WQ_{it}}{P_{it}} \end{aligned} \quad (D8)$$

Thus

$$\frac{\partial \ln(P_{it})}{\partial WQ_{it}} * WQ_{it} = elast_i \quad (D9)$$