Appendix C: Hypothesis tests on coefficients of model in eq. (5b)

We performed multiple statistical tests on the coefficients of the LGA interactions δ_k^{LGSA} from the model in eq. (5b).

First, we test the hypothesis that all coefficients of treatment interaction terms δ_k^{LGSA} with k = 2, 3, 4, 5, 6-8, and 9+ participants are equal to zero based on F-test. Based on the results, we reject the null hypothesis that the respective coefficients are jointly equal to zero at conventional levels of statistical significance (see Table C1).

Second, we conduct a t-test to check whether the coefficients δ_k^{LGSA} are pairwise ($\delta_k^{LGSA} = \delta_l^{LGSA}$ with $k \neq l$) significantly different from each other. Based on the results, we find that the coefficient estimates for LGSA auctions with 2, 3, 4, and 5 participants differ significantly from the coefficient for LGSA auctions with 9+ participants at least at the 5% level, and that the coefficients for LGSA auctions with 3, 4, and 5 participants differ significantly from the coefficient for 6-8 participants at least at the 10% level.We also find that the coefficients for 2, 3, 4, and 5 participants do not differ significantly from each other at conventional levels of statistical significance (see Table C2).

Third, we conduct multivariate one-sided tests (Wolak 1987; Silvapulle and Sen 2001) to test whether the coefficients for 2, 3, 4, and 5 participants are smaller than (<) the coefficients for 6-8 and 9+ participants, respectively. Following Vanbrabant and Rosseel (2020), we implement a two-stage testing procedure. For δ_{9+}^{LGSA}, based on an F-test, the *first step* tests

$$H_0\colon \delta_2^{LGSA} \leq \delta_{9+}^{LGSA}; \delta_3^{LGSA} \leq \delta_{9+}^{LGSA}; \delta_4^{LGSA} \leq \delta_{9+}^{LGSA}; \delta_5^{LGSA} \leq \delta_{9+}^{LGSA} \ versus$$
$$H_1\colon at\ least\ one\ inequality\ is\ violated.$$

Not rejecting H_0 would allow equalities; in this case, the *second step* tests

$$H_0\colon \delta_2^{LGSA}; \delta_3^{LGSA}; \delta_4^{LGSA}; \delta_5^{LGSA} = \delta_{9+}^{LGSA} \ versus$$
$$H_1\colon at\ least\ one\ equality\ is\ violated.$$

Test results from the *first step* cannot reject the null hypothesis that all restrictions hold ($\bar{F}^1 = 0, p = 1$, see Table C3). Based on the results of the *second step*, we reject the null hypothesis that all restrictions are equalities ($\bar{F}^2 = 15.448, p = 0.0017$). For δ_{6-8}^{LGSA} , we conduct the test analogously (see Table C4).

Fourth, following Vanbrabant and Rosseel (2020), we test whether the data support the order-constrained hypothesis that the price differences between the sellers are decreasing with an increasing number of participants, i.e., $\delta_2^{LGSA} < \delta_3^{LGSA} < \delta_4^{LGSA} < \delta_5^{LGSA} < \delta_{6-8}^{LGSA} < \delta_{9+}^{LGSA}$. The *first step* tests

$$H_0\colon \delta_2^{LGSA} \leq \delta_3^{LGSA} \leq \delta_4^{LGSA} \leq \delta_5^{LGSA} \leq \delta_{6-8}^{LGSA} \leq \delta_{9+}^{LGSA} \ versus$$
$$H_1\colon at\ least\ one\ inequality\ is\ violated.$$

Not rejecting H_0 would allow equalities; in this case, the *second step* tests

$$H_0: \delta_2^{LGSA} = \delta_3^{LGSA} = \delta_4^{LGSA} = \delta_5^{LGSA} = \delta_{6-8}^{LGSA} = \delta_{9+}^{LGSA} \ versus$$
$$H_1: at\ least\ one\ equality\ is\ violated.$$

The restricted model adjusts for the violation of the order-constrained hypothesis by forcing δ_2^{LGSA} to be smaller than or equal to δ_3^{LGSA}. The results from the *first step* show that the order-constrained hypothesis is not rejected in favour of the unconstrained hypothesis ($\bar{F}^1 = 2.6977\ p = 0.5105$, see Table C5). Based on the results from the *second step* ($\bar{F}^2 = 18.497, p = 0.0001$), we can reject the null hypothesis that all restrictions are equalities, i.e., the data support the constraints and impose only minor changes on the coefficient estimates of the model in eq. (5b).

References

Silvapulle, Mervyn J., and Pranab K. Sen. 2001. *Constrained Statistical Inference.* Hoboken, NJ, USA: John Wiley & Sons, Inc.

Vanbrabant, Leonard, and Yves Rosseel. 2020. “An Introduction to Restriktor: Evaluating Infromative Hypotheses for Linear Models.” In *Small Sample Size Solutions: A Guide for Applied Researchers and Practitioners*, 157–72. European Association of Methodology series. London, New York: Routledge.

White, Halbert. 1980. “A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity.” *Econometrica* 48 (4): 817–38.

Wolak, Frank A. 1987. “An Exact Test for Multiple Inequality and Equality Constraints in the Linear Regression Model.” *Journal of the American Statistical Association* 82 (399): 782–93.

Table C1 F-test

Hypothesis:

H_0: $\delta_2^{LGSA} = \delta_3^{LGSA} = \delta_4^{LGSA} = \delta_5^{LGSA} = \delta_{6-8}^{LGSA} = \delta_{9+}^{LGSA} = 0$

H_1: $\delta_k^{LGSA} \neq 0$ for at least one $k = 2$, 3, 4, 5, 6-8, 9+

	Res. DF	DF	F-Value	Pr(>F)
1	1468			
2	1462	6	20.209***	2.2e-16

Note: Asterisks indicate *p = <0.1; **p = <0.05; ***p = <0.01. F-test is based on robust standard errors according to White (1980).

Table C2 Two-sided t-test H_0: $\widehat{\delta_k^{LGSA}} = \widehat{\delta_l^{LGSA}}$ based on model (5b)

k \ l	δ_2^{LGSA}	δ_3^{LGSA}	δ_4^{LGSA}	δ_5^{LGSA}	δ_{6-8}^{LGSA}	δ_{9+}^{LGSA}
δ_2^{LGSA}	-	-1.379 (0.168)	-1.121 (0.262)	-0.229 (0.819)	-1.283 (0.200)	-2.198 ** (0.028)
δ_3^{LGSA}	-1.379 (0.168)	-	-0.195 (0.845)	-1.302 (0.193)	-3.451 *** (0.001)	-4.457 *** (0.000)
δ_4^{LGSA}	-1.121 (0.262)	-0.195 (0.845)	-	-1.004 (0.315)	-2.863 *** (0.004)	-3.814 *** (0.000)
δ_5^{LGSA}	-0.229 (0.819)	-1.302 (0.193)	-1.004 (0.315)	-	-1.763 * (0.078)	-2.774 *** (0.006)
δ_{6-8}^{LGSA}	-1.283 (0.200)	-3.451 *** (0.001)	-2.863 *** (0.004)	-1.763 * (0.078)	-	-1.232 (0.218)
δ_{9+}^{LGSA}	-2.198 ** (0.028)	-4.457 *** (0.000)	-3.814 *** (0.000)	-2.774 *** (0.006)	-1.232 (0.218)	-

Note: Value of *t*-statistic reported with the respective p-value in parentheses. Asterisks indicate *p = <0.1; **p = <0.05; ***p = <0.01.T-tests are based on robust standard errors according to White (1980).

Table C3 Test results for inequality restrictions $\delta_2^{LGSA}; \delta_3^{LGSA}; \delta_4^{LGSA}; \delta_5^{LGSA} < \delta_{9+}^{LGSA}$

Step 1	*Step 2*
H_0: All restrictions hold in the population $\delta_2^{LGSA} \leq \delta_{9+}^{LGSA}$; $\delta_3^{LGSA} \leq \delta_{9+}^{LGSA}$; $\delta_4^{LGSA} \leq \delta_{9+}^{LGSA}$; $\delta_5^{LGSA} \leq \delta_{9+}^{LGSA}$ H_1: At least one inequality is violated	H_0: All restrictions are equalities $\delta_2^{LGSA} = \delta_{9+}^{LGSA}$; $\delta_3^{LGSA} = \delta_{9+}^{LGSA}$; $\delta_4^{LGSA} = \delta_{9+}^{LGSA}$; $\delta_5^{LGSA} = \delta_{9+}^{LGSA}$ H_1: At least one equality is violated
F-test statistic p-value 0.0000 1.0000	F-test statistic p-value 15.448*** 0.0017

Note: Asterisks indicate *p = <0.1; **p = <0.05; ***p = <0.01. F-tests are based on robust standard errors according to White (1980).

Table C4 Test results for inequality restrictions $\delta_2^{LGSA}; \delta_3^{LGSA}; \delta_4^{LGSA}; \delta_5^{LGSA} < \delta_{6-8}^{LGSA}$

Step 1		*Step 2*	
H_0: All restrictions hold in the population $\delta_2^{LGSA} \leq \delta_{9+}^{LGSA}$; $\delta_3^{LGSA} \leq \delta_{9+}^{LGSA}$; $\delta_4^{LGSA} \leq \delta_{9+}^{LGSA}$; $\delta_5^{LGSA} \leq \delta_{9+}^{LGSA}$ H_1: At least one inequality is violated		H_0: All restrictions are equalities $\delta_2^{LGSA} = \delta_{9+}^{LGSA}$; $\delta_3^{LGSA} = \delta_{9+}^{LGSA}$; $\delta_4^{LGSA} = \delta_{9+}^{LGSA}$; $\delta_5^{LGSA} = \delta_{9+}^{LGSA}$ H_1: At least one equality is violated	
F-test statistic	p-value	F-test statistic	p-value
0.0000	1.0000	12.880***	0.0048

Note: Asterisks indicate *p = <0.1; **p = <0.05; ***p = <0.01. F-tests are based on robust standard errors according to White (1980).

Table C5 Test results for inequality restrictions $\delta_2^{LGSA} < \delta_3^{LGSA} < \delta_4^{LGSA} < \delta_5^{LGSA} < \delta_{6-8}^{LGSA} < \delta_{9+}^{LGSA}$

Step 1		*Step 2*	
H_0: All restrictions hold in the population $\delta_2^{LGSA} \leq \delta_3^{LGSA}$; $\delta_3^{LGSA} \leq \delta_4^{LGSA}$; $\delta_4^{LGSA} \leq \delta_5^{LGSA}$; $\delta_5^{LGSA} \leq \delta_{6-8}^{LGSA}$ $\delta_{6-8}^{LGSA} \leq \delta_{9+}^{LGSA}$ H_1: At least one inequality is violated		H_0: All restrictions are equalities $\delta_2^{LGSA} = \delta_3^{LGSA}$; $\delta_3^{LGSA} = \delta_4^{LGSA}$; $\delta_4^{LGSA} = \delta_5^{LGSA}$; $\delta_5^{LGSA} = \delta_{6-8}^{LGSA}$ $\delta_{6-8}^{LGSA} = \delta_{9+}^{LGSA}$ H_1: At least one equality is violated	
F-test statistic	p-value	F-test statistic	p-value
2.6977	0.5105	18.4971***	0.0001

Note: Asterisks indicate *p = <0.1; **p = <0.05; ***p = <0.01. F-tests based on robust standard errors according to White (1980).